We have seen in the presentation that the constrained optimization problem

$$\min_{\beta_0,\beta} \frac{1}{2}||\beta||^2 + \gamma \sum_i \xi_i$$
$$\text{s.t. } \xi_i \geq 0, y_i(h^T(x)\beta + \beta_0) \geq 1 - \xi_i$$

which is solved by the SVM algorithm is equivalent to the minimization problem

$$\min_{\beta_0,\beta} \sum_i [1 - y_i(h^T(x)\beta + \beta_0)]_+ + \lambda||\beta||^2$$

Here ξ_i are the so called slack-variables; h is a vector of basis functions; $[\ldots]_+$ denotes the positive part.
The above text is just to recall the problem; the only thing you need for the exercises is that the classification boundary from a SVM is of the form $h^T(x)\beta + \beta_0 = 0$ for a set of basis functions that are determined by the choice of a kernel. We will compute this in a simple example.

1. The SVM algorithm bases its calculations on a kernel which in turn determines the basis functions h_i that are used to fit the classification boundary. Consider the kernel in 2 dimensions $K(x, x') = (1 + x_1 x_1' + x_2 x_2')^2$. We want to express K as a sum of M basis functions $h_1, \ldots, h_M$ such that $K(x, x') = \sum_{i=1}^M h_i(x)h_i(x')$. Find the basis functions h_i. (Hint: $M = 6$)

2. Now consider the classification boundary $h^T(x)\beta + \beta_0 = 0$ where h is the vector of the functions h_i. Can you visualize the boundary for some values of β ? Hint: you may want to set up a grid of values via the command `expand.grid` for two one-dimensional sequences. Then use `apply`with the appropriate function and stick this into a matrix that has `nrow = length(sequence)`; finally get the contour via `contour(...,levels=0)`. I created random β entries, but there might be other ways to do this. Try a few random β vectors and see what you get.