Simple Homework (Wei's Presentation).

Sample $\mathbf{X}_1, \cdots, \mathbf{X}_n$ i.i.d $\sim (\mathbf{0}, \Gamma)$.

$\hat{\mathbf{u}}$: the estimate of the leading eigenvector of Γ based on the sample of size

n, i.e.

$$\mathbf{u} = \arg\max_{||\mathbf{u}||=1} \ \mathbf{u}^T \ \mathbf{G} \ \mathbf{u}$$

where G is the sample covariance matrix.

And

$$CV(\lambda) = \sum_{i=1}^{n} \mathbf{E}||\mathbf{X}_i - \{\mathbf{X}_i^T \ \mathbf{u}_1^{(-i)}(\lambda)\}\mathbf{u}_1^{(-i)}(\lambda)||^2.$$

$\mathbf{X}$ indept of $\mathbf{X}_1, \cdots, \mathbf{X}_n$.

Define the mean-square prediction error

$$L_n(\hat{\mathbf{u}}) = \mathbf{E}_{\mathbf{X}}||\mathbf{X} - (\mathbf{X}^T \ \hat{\mathbf{u}})\hat{\mathbf{u}}||^2$$

and

$$R_n(\lambda) = \mathbf{E}[L_n\{\hat{\mathbf{u}}_1(\lambda)\}]$$

Show that

$$\mathbf{E}n^{-1}\mathbf{CV}(\lambda) = R_{n-1}(\lambda).$$