

Solutions to STAT 200 Midterm Exam I

Problem 1. Answers: T F F T F

Problem 2. Answers: c, d, a, b, b, a, b, b, a, c

Problem 3. (b). Based on the given information, we have $b = -0.03$, $0.42 = a - 0.03 \times 5$. So $a = 0.42 + 0.15 = 0.57$, and the regression line is given by

$$y = 0.57 - 0.03x.$$

(c). The slope changes sign (from negative to positive), and the intercept changes substantially, so the 5th observation is influential. The dotted line is more reliable for prediction.

(d). $r^2 = (-0.75)^2 = 56.25\%$.

If we remove the last observation, the number in (i) will increase.

(e). Answers: T T F T

Problem 4. (a) Let X be the GRE score. We have

$$500 < X < 700 \quad \rightarrow \quad \frac{500 - 560}{100} < Z < \frac{700 - 560}{100} \quad \rightarrow \quad -0.6 < Z < 1.4.$$

From Table A, we have $0.9192 - 0.2743 = 0.6449 = 64.49\%$.

(b). $\frac{X-560}{100} \geq 1.28 \quad \rightarrow \quad X \geq 560 + 128 = 688$.

(c). Let Y be the number of applicants with GRE scores higher than 750. Then

$$p = P(X > 750) = P\left(Z > \frac{750 - 560}{100}\right) = P(Z > 1.9) = 0.0287.$$

$$\begin{aligned} P(Y \geq 1) &= 0.90 \quad \rightarrow \quad P(Y = 0) = 0.10 \\ &\rightarrow 1 \times 0.0287^0 \times (1 - 0.0287)^n = 0.10 \\ &\rightarrow 0.9713^n = 0.10 \\ &\rightarrow n = \frac{\ln(0.10)}{\ln(0.9713)} = 79.07 \approx 80. \end{aligned}$$

Problem 5. (a)

$$P(X = 0) = \frac{\binom{3}{2}}{\binom{5}{2}} = 0.3,$$

$$P(X = 1) = \frac{\binom{2}{1} \times \binom{3}{1}}{\binom{5}{2}} = 0.6,$$

$$P(X = 2) = \frac{\binom{2}{2}}{\binom{5}{2}} = 0.1.$$

(b)

$$E(X) = 0 \times 0.3 + 1 \times 0.6 + 2 \times 0.1 = 0.8$$

$$\sigma = \sqrt{V(X)} = \sqrt{(0 - 0.8)^2 \times 0.3 + (1 - 0.8)^2 \times 0.6 + (2 - 0.8)^2 \times 0.1} = 0.6$$