Solutions to Sample STAT 200 Midterm Exam II

1. F, T, T, T, T.

2 (16 pts). ii, ii, iv, iv.

3. (a) Let X be the content of a randomly selected bottle. Then $X \sim N(298, 3)$. We have

$$\bar{X} = \sum_{i=1}^{4} X_i \sim N(298, \frac{3}{2}) = N(298, 1.5).$$

Thus

$$\begin{aligned} P(295 < \bar{X} < 299) &= P(\frac{295-298}{1.5} < Z < \frac{299-298}{1.5}) \\ &= P(-2 < Z < 0.67) \\ &= 0.7486 - 0.0228 = 0.726. \end{aligned}$$

(b) (10 pts) Let p be the probability that a randomly selected bottle is good. Then

$$p = P(X \geq 300) = P(Z \geq \frac{300-298}{3}) = P(Z > 0.67) = 0.2514.$$

Let Y be the number of good bottles among the 100 bottles. Then Y follows a binomial distribution with $n = 100$ and $p = 0.2514$, and

$$E(Y) = np = 100 \times 0.2514, \qquad V(Y) = np(1-p) = 100 \times 0.2514 \times (1 - 0.2514).$$

Thus, based on the Central Limit Theorem, we have

$$\begin{aligned} P(Y \leq 80) &\approx P(Z < \frac{80 - 100 \times 0.2514}{\sqrt{100 \times 0.2514 \times (1-0.2514)}}) \\ &= P(Z < 12.70) = 1. \end{aligned}$$

4. (a) Let μ be the population change in stress level. We want to test

$$H_0 : \mu = 0 \qquad vs. \qquad H_a : \mu < 0$$

Test statistic

$$t = \frac{\bar{x} - 0}{s/\sqrt{n}} = \frac{-1.2}{2.8/5} = -2.143.$$

p-value $= P(t(24) < -2.143)$. We have $0.02 <$ p-value < 0.025.

Thus, there is strong evidence that the treatment effect decreases over time.

(b) The 95% C.I. for μ is $\bar{x} \pm z^*\sigma/\sqrt{n}$.
We want length $= 2z^*\sigma/\sqrt{n} < 1$, so we have

$$n > (2z^*\sigma)^2 = (2 \times 1.96 \times 1.5)^2 = 35.$$

5. (a) $\bar{x} \pm 1.96 \times 10/\sqrt{25} = 136 \pm 3.92$
So a 95% confidence interval is (132.08, 139.92).

(b)We want to test the following hypotheses

$$H_0 : \mu = 128 \qquad vs \qquad H_a : \mu \neq 128.$$

We have the following information

$$\alpha = 0.05, \quad \bar{x} = 136, \quad \sigma = 10, \quad n = 25, \quad z^* = 1.96.$$

The test statistic is given by

$$z = \frac{\bar{x} - \mu_0}{\sigma}\sqrt{n} = \frac{136 - 128}{10} \times \sqrt{25} = 4.0.$$

P-value $= P(|Z| > 4.0) < 0.001$, which is very small, so we have strong evidence against the claim that executives' blood pressures are the same as the national mean.

Since p-value < 0.05 (or $|z| = 4.0 > z^* = 1.96$), we reject H_0 at 5% level, and conclude that the mean blood pressure of executives may not equal to the national mean.

(c) (i) We want to test the following hypotheses

$$H_0 : \mu = 128 \qquad vs \qquad H_a : \mu > 128$$

(Note that H_a is different from (b)!) Based on results in (a), we have

$$p - value \ = P(Z > 4.0) < 0.001.$$

Therefore, there is very strong evidence against the claim.

(ii) If executives' blood pressures equal to the national mean (the claim), the chance of observing the above sample with $z = 4.0$ or more extreme is less than 0.001. This implies that the claim may not be true and thus the executives' blood pressures should be higher than the national mean.